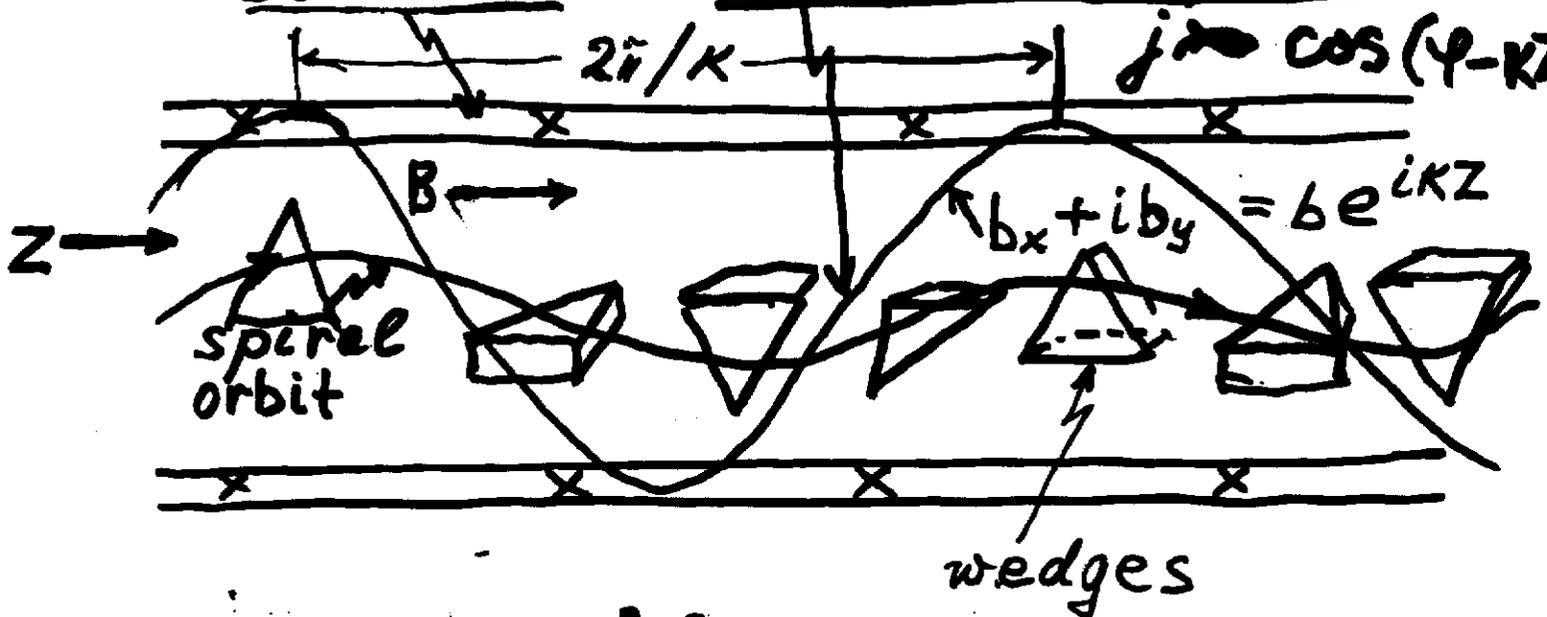


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1. General arrangement

- Solenoid + helical dipole

$$b = b_0 \left(1 + \frac{1}{4} K^2 \rho^2 + \dots \right); \quad K \sim K_c \equiv \frac{eB}{pc}$$

• quad and sext can be added (same period)

• typical values: $B = (6 \rightarrow 1 \rightarrow 20) \text{ T}$
 $b \geq 2 \text{ kG}$

$$\alpha \equiv \frac{b}{B} = (3 \rightarrow 20 \rightarrow 3)\%$$

Why Spiral Transport?

- dispersion introduced
- conservative focusing
- 3-dim cooling appears
- field can vary along beam pass

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Trajectories

- spiral orbit radius $a(\rho)$:

$$a(\rho) \approx \lambda \frac{a}{1 - \frac{v_{pc}}{c}}; \quad \lambda \equiv k^{-1}$$

$$\left| 1 - \frac{v_{pc}}{c} \right| \gg \alpha^{2/3}, \quad \theta_E^2$$

Transverse position expansion:

$$\vec{\rho} = \vec{a}(\rho, z) + \vec{\rho}_c + \vec{d}$$

dispersed
orbits
frequency \underline{k}

cyclotron
oscillation,
frequency k_c

Larmor
center,
slow
drift;

$$\rho_c = \frac{\rho_{pc}}{eB}$$

cyclotron
emittance

$$\epsilon_c = \frac{\langle \rho_c^2 \rangle}{m e B}$$

drift
emittance

$$\epsilon_d = \frac{\langle d^2 \rangle}{m c^2 / e B}$$

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- Linear stability is established over a large Δp ($\sim 100\%$ of particles)
- Expectation regarding the non-linear effects at $\delta_\varepsilon \ll \bar{x}$ (regardless to KQ value):
 - tune shift of cyclotron mode, $\Delta K_c \approx K_c \theta_{sp}^2 / D$
 - slow drift rotation, $KQ_d \approx K (\Delta K_c / K)^2$
 \uparrow
 (Q-value)

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2. Dispersion \propto Stability

Dispersion parameter:

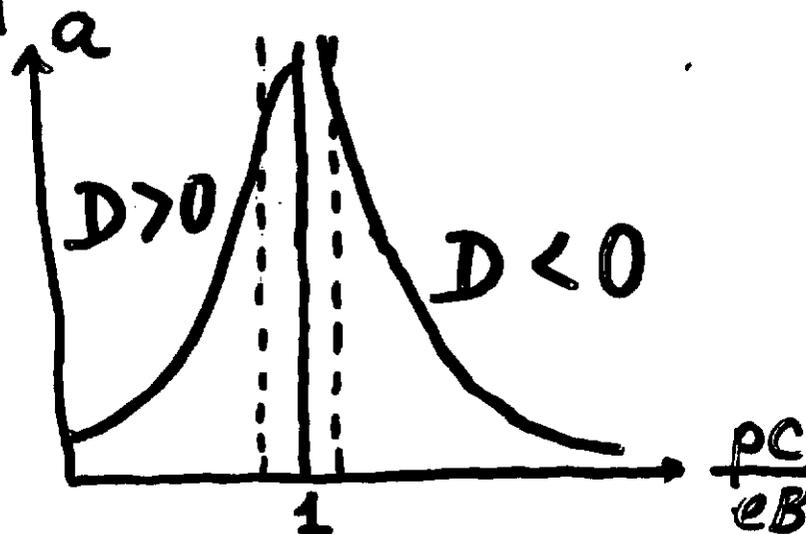
$$D \equiv \frac{p}{a} \frac{da}{dp} \approx \frac{1}{\frac{eB}{pc} - 1} \equiv \frac{1}{q}$$

- D should not change the sign
- D should not be too large:

$$D < (B/b)^{2/3}, \quad 1/\theta_E^2$$

- Two areas of Stability:

$$D > 0, \quad \frac{cp_{\max}}{eB} < \lambda \quad \text{and} \quad D < 0, \quad \frac{cp_{\min}}{eB} > \lambda$$



3. Cooling Rates

3.1. Energy spread rate

$$\langle E' \rangle = \text{const} \langle n(\vec{p}, z) \rangle$$

$$(\vec{p} = a(E, z) + \vec{a} + \vec{p}_c)$$

Model:

$$\langle n \rangle = \langle n \rangle_0 + (\nabla n) \frac{\vec{a} \cdot \vec{p}}{a}$$

$$\delta \gamma' = -\Lambda_\gamma \delta \gamma,$$

$$\Lambda_\gamma = \Lambda_0 \left(-\frac{1}{\gamma^2} + \frac{1}{2} \frac{a \nabla n}{\langle n \rangle_0} \right) / \beta^2$$

$$(\Lambda_0 = -2\gamma'/\gamma)$$

3.2. Cyclotron rate: $\epsilon'_c = -\Lambda_c \epsilon_c$

$$\Lambda_c = \Lambda_0 \left(1 - \frac{\kappa}{2\kappa_c} \frac{a \nabla n}{n} \right) / \beta^2$$

3.3. Drift emittance rate

$$\vec{d}' = -\frac{1}{2} \Lambda_d \vec{d};$$

$$\Lambda_d = -\frac{\kappa}{\kappa_c} \frac{a \nabla n}{n} \cdot \frac{1}{2} \Lambda_0 / \beta^2$$

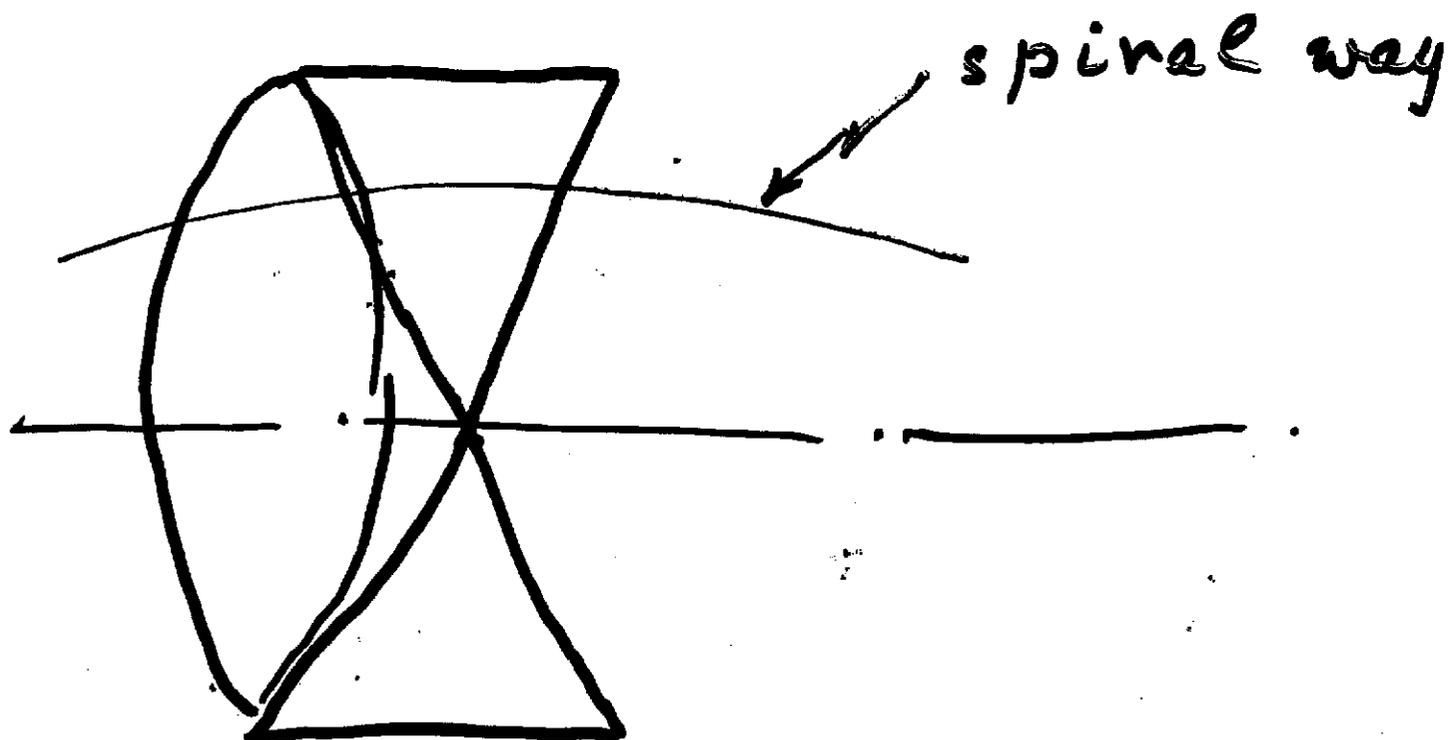
$$\vec{d}'_{\text{instant}} = \frac{\vec{F}_\perp \times \vec{B}}{e B^2}$$

Condition:

$$1 \ll 9$$

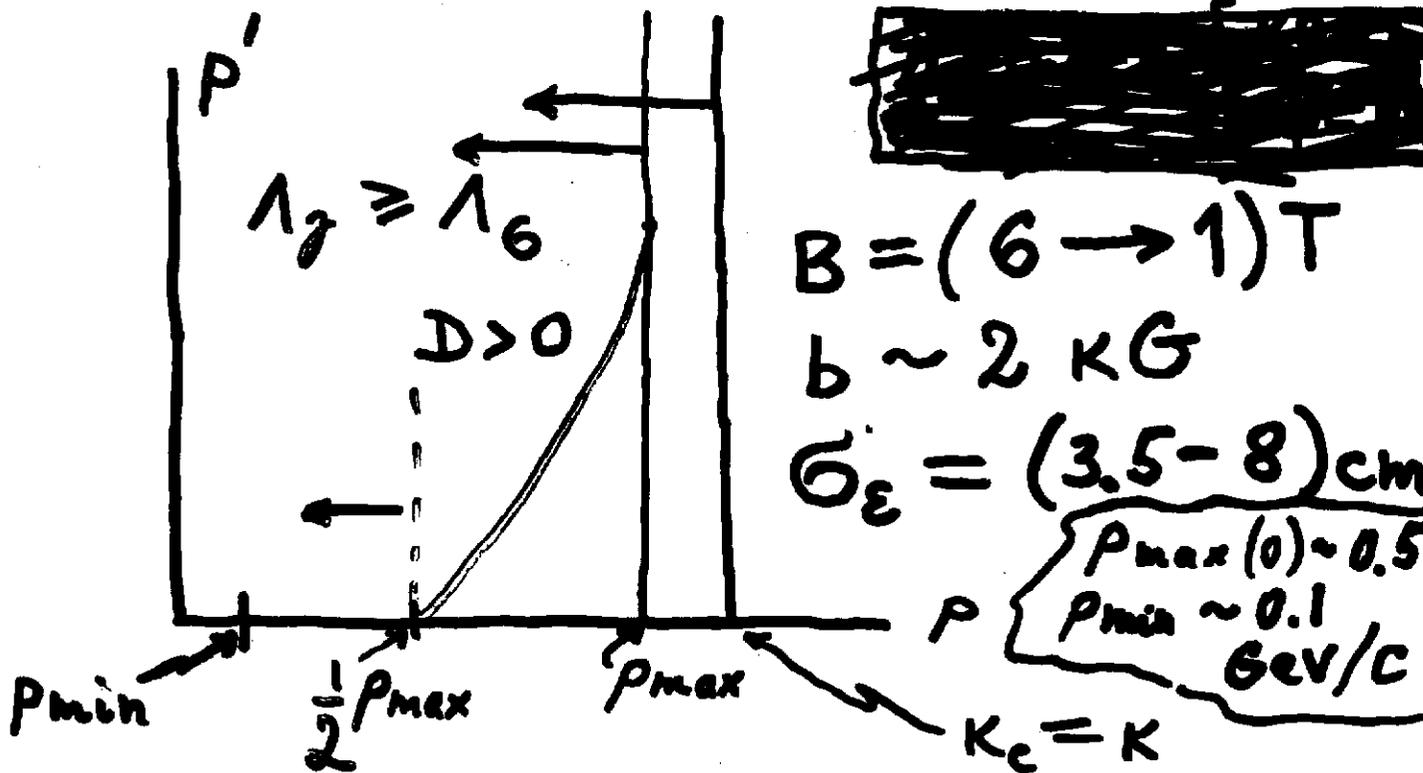
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axial wedges option (A. Sessler)



$$\begin{aligned}
 \langle n \rangle(\rho) &= \langle n \rangle (|\vec{\rho}|) = \\
 &\approx \langle n \rangle \left[a(\rho) + \frac{\vec{a}}{a} (\vec{d} + \vec{\rho}_c) + \dots \right] \\
 &= \langle n \rangle (a(\rho)) + \nabla n \cdot \frac{\vec{a}}{a} (\vec{d} + \vec{\rho}_c)
 \end{aligned}$$

4. Sweeping a Large Initial Energy Spread (no RF/acceleration)



Sweeping regime: $n(p) = n_0 \left[\frac{\kappa p - 2\alpha}{\kappa p - \alpha} \right]^2$
 $\frac{\kappa p_{max}(z)}{eB(z)} \approx \text{const}, \quad \nabla n > 0$
 $B' \approx \text{const}$

$a_{max}(z) = \text{const} \cdot \sigma_E$

- $|\lambda_d| \approx |\lambda_c| \ll \lambda_r$; (factor $\sim 1/4$)
- $(a_{max}/\sigma_E) \approx 3$; $q = (0.35 \rightarrow 0.13)$
- $\lambda_d < 0, \lambda_c < 0$

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5. RF capture

- Introduce sextupole helix to compensate for slippage aberration

(cancel $\frac{\partial V_H}{\partial \theta_c^2}$)

- RF option:

use TEM $\lambda/4$
resonators

(?)

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6. Regular Cooling at $\Delta p \ll p$

- regime $D < 0$, $\langle \frac{\partial n}{\partial p} \rangle < 0$

- equalize decrements
(option)

$$\Lambda_z = \Lambda_c = \Lambda_d = \frac{2}{3} \Lambda_6$$

7. Equilibrium

- optimization to min \mathcal{E}_6 :

$$\Lambda_c = \Lambda_d = \Lambda_z = \frac{2}{3} \Lambda_6$$

8. Reverse emittance

exchange (post-equilibrium)

Regime:

$$D < 0; \quad \frac{\Delta n}{n} \gg 1$$

$$(\mathcal{E}_\perp)_{\min} = \sqrt{(\mathcal{E}_6)_{\min} / (\Delta p)_{\max}}$$

(if any benefits...)

- // -

9. Issues

• There are many...

technique issues, at least

• Best wedges, for instance...

• Quality limits of
cooling process

(especially sweeping)

• Simulations accompanied
by analytical efforts
are needed!